Teaching briefs... An Update On Fractals

by Elyse Magram

In the summer of 1991 I attended a one week program on "Chaos, Fractals and Iteration" at Rutgers University. The program was part of the ongoing Leadership Program in Discrete Mathematics, to which the previous year's "graduates" were invited back to learn about fractal geometry. I was particularly enthusiastic about participating because I had heard about fractals the prior summer, had done extensive research on my own, and had introduced fractals to four of my classes with great enthusiasm. I wrote about this in the previous issue of this Newsletter.

Dr. Terry Perciante, professor of mathematics at Wheaton College in Illinois, was the dynamic force behind the week-long program. He introduced us to the material from the new NCTM book "Fractals for the Classroom" of which he is a coauthor.

The course exposed me to such an enormous amount of mathematics, and to so much correlation of math with fractals, that I couldn't wait to teach the topic.

This year, I used worksheets from "Fractals for the Classroom" on the Pascal triangle and cellular automata to introduce the relationship between modular arithmetic, the Pascal triangle, and the Sierpinski triangle. We then did the "chaos game" described on the right, and viewed programmed computer pictures of the Sierpinski triangle and the Sierpinski carpet. We discussed "why" this worked, using probability and area intersections. The students found all of the geometric, algebraic, probability, and arithmetic correlations amazing and thought-provoking.

Each student was responsible for doing one "colored" fractal, and urged to color as creatively as possible. We now have more than one hundred fifty of these. We have "fractal monsters" with faces and secret messages in the Hilbert curve, a Koch curve with "West," the name of our school, and a 3-D Menger Sponge of foam core. One student oil-painted her Koch snowflake (see issue #1, page 4).

We discussed the perimeter, and then the area, of the nth level of the Sierpinski triangle and carpet. I was amazed at the participation and the logic displayed by the students in finding these "limiting terms."

I spent two and one-half weeks prior to the holidays on this topic in Precalculus and two weeks in my slow, non-regents applications class. In the slower class, we didn't discuss limits.

I felt that the material provided was fantastic. The gridded worksheets in the NCTM book in the program saved a lot of time that I had previously used to measure segments and angles. Students were extremely interested in the topic, saw many applications and really "got into it." I was amused when one of my students told me how thrilled he had been to see his fractal image shaving that morning. He had rigged up an extra mirror so that his image was reflected to infinity.■

The Chaos Game

The chaos game magically collects randomly generated points into the familiar, highly structural Sierpinski triangle.

Start with three fixed points L(left), R(right) and T(top), vertices of an equilateral triangle, and a random point S within the triangle. Choose L, R, or T randomly and move halfway from S toward that vertex to get the point S_1. (For example, roll a die and assume a roll of 1 or 2 corresponds to vertex T, a roll of 3 or 4 corresponds to vertex L, and a roll of 5 or 6 corresponds to vertex R.) Choose L, R, or T randomly and move halfway from S_1 to the vertex to get the point S_2. Repeat this process, so that S_{n+1} is the midpoint of the line segment from S_n to a randomly chosen vertex L, R, or T.

In the first diagram, S_1 is halfway from S to R, S_2 is halfway from S_1 to T, S_3 is halfway from S_2 to R, and S_4 is halfway from S_3 to L. The midpoints S_1, S_2, S_3,... fill up the shaded portions of the second diagram, forming what is called (after infinite iteration) the Sierpinski triangle.

To enhance the image, ask different students to play the chaos game on transparencies marked only with identically placed triangles, and stack the resulting transparencies on the overhead projector.

You can involve geometry, algebra, probability, limits, and other topics in your classroom discussion of the chaos game.■